Effects of Superconducting Losses in Pulse Propagation on Microstrip Lines

O. R. Baiocchi, K.-S. Kong, H. Ling, and T. Itoh

Abstract—**An analysis of the effect of losses in the propagation of pulses on superconducting microstrip lines is presented. It is based on the phenomenological equivalence method (PEM) and the two-fluid model of superconductivity to calculate the propagation characteristics of the superconducting line. For most practical situations, it is shown that these losses do not introduce phase distortion, and the attenuation constant can be approximated by a quadratic expression of the frequency. Therefore, the effect of attenuation can be easily evaluated through a simple equivalent filter.**

Introduction

The phenomenological equivalence method [1] provides a simple way to evaluate the losses in the superconducting microstrip line, and can be easily extended to other structures [2]. Its main advantage is to include both the field penetration and a geometric factor into the formulation; expressions for the surface impedance, per-unit impedance and propagation constants are obtained in terms of these factors. For the range of frequencies below the energy-gap of the superconducting material, the two-fluid model of superconductivity is appropriately used with this method. The dependence of the attenuation constant with the frequency under this assumption is approximately quadratic [3]–[5].

In this letter, we show that this dependence can also be derived from the PEM formulation with the advantage of providing an expression that contains both the geometric factor and the penetration depth. It is also shown how to model the propagation of a pulse in the microstrip line, using a simple equivalent filter or transfer function. For a Gaussian-shaped pulse, the distortion can be evaluated analytically. It is also shown that the effect of these losses on the phase distortion within the limits of the approximations, is too small to be considered.

Conductor Loss Calculation

We use the two-fluid model to describe the superconductivity, in which the complex conductivity is given by

$$\sigma_{sc} = \sigma_1 - j\sigma_2 \tag{1a}$$

$$\sigma_1 = \sigma_n (T/Tc)^4 \tag{1b}$$

$$\sigma_2 = \left(1 - (T/T_c)^4\right)/(\lambda^2 \omega\mu) \tag{1c}$$

where σ_n, λ and T_c are the normal conductivity near the critical temperature, zero temperature penetration depth and critical temperature of the superconducting material, respectively. This complex conductivity is used to calculate the surface impedance of the superconductor:

$$Z_S = \left[j\omega\mu / (\sigma_1 - j\sigma_2) \right]^{1/2} \tag{2}$$

and the per-unit series impedance introduced by the superconductor.

$$Z_i = R_i + jX_i = Z_S G \coth\left[(\sigma_1 - j\sigma_2) Z_S A G \right] \tag{3}$$

where G is a numerical factor depending only on the geometry of the line, and A, the strip conductor cross-section [1].

Once Z_i is calculated, the attenuation and propagation constant can be found using the transmission line equivalent circuit.

Quadratic Approximation

Let us introduce the parameter $q(\omega)$, which is in fact the ratio between the real and imaginary parts of the complex conductivity:

$$q(\omega) = \mathrm{Re}\left[\sigma_{sc}\right] / \mathrm{Im}\left[\sigma_{sc}\right] = (\sigma_n \lambda^2 \omega\mu\theta)/(1-\theta) \tag{4}$$

where $\theta = (T/T_c)^4$. For most cases of interest, and within the limits of frequency that allow the use of the two-fluid model, this factor is much smaller than one. Consequently, a binomial expansion for Z_s can be obtained, where the terms of order two or higher on $q(\omega)$ are neglected. Combining these expressions into (3), and separating the real and imaginary parts of Z_i, we obtain:

$$R_i = \frac{\omega^2 \theta G}{(1-\theta)^{3/2}} \frac{\sigma_n}{2} \mu^2 \lambda^3 \cdot \left[\coth \frac{AG\sqrt{1-\theta}}{\lambda} + \frac{AG\sqrt{1-\theta}}{\lambda} \operatorname{cosech}^2 \frac{AG\sqrt{1-\theta}}{\lambda} \right] \tag{5a}$$

$$X_i = \frac{\omega\mu\lambda G}{\sqrt{1-\theta}} \coth \frac{AG\sqrt{1-\theta}}{\lambda} - \omega^3 \frac{\theta^2 G^2}{(1-\theta)^2} \left(\frac{\sigma_n}{2}\right)^2 \mu^3 \lambda^4 A \operatorname{cosech}^2 \frac{AG\sqrt{1-\theta}}{\lambda}. \tag{5b}$$

These expressions show that the real part of Z_i (and therefore, the attenuation constant) is proportional to the square of the frequency, and the imaginary part of it (and so the propagation constant) is predominantly a linear function of the frequency, as reported in many references. Our expressions differ from them, however, in the sense that the geometric factor G of the structure and the penetration depth are included in the expressions.

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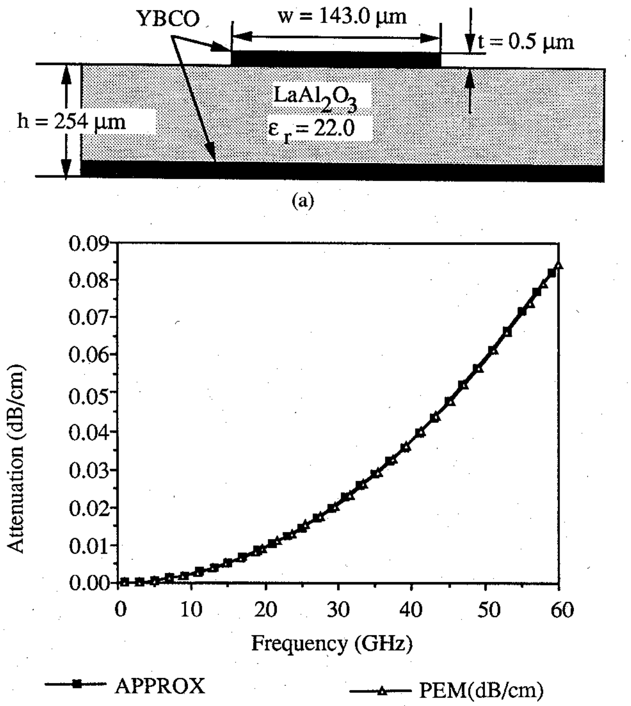

Fig. 1. Comparison of attenuations of superconducting microstrip line obtained by approximation and PEM (at 77 K). (a) High T_c superconductor microstrip line structure. YBCO: T_c = 92.5 K, σ_n = 1.7 S/μm, λ = 0.3 μm. (b) Attenuation due to conductor loss versus the frequency.

Modeling the Effects of Losses

From (5a), and using the simple transmission line equivalent circuit, the attenuation constant can be obtained by

$$\alpha \cong \frac{\mathrm{Re}\left[Z_i\right]}{2.0 Z_0} = \frac{\theta}{(1-\theta)^{3/2}} \frac{G\sigma_n}{4.0 Z_0} \mu^2 \lambda^3 \cdot \left[\coth \frac{AG\sqrt{1-\theta}}{\lambda} + \frac{AG\sqrt{1-\theta}}{\lambda} \operatorname{cosech}^2 \frac{AG\sqrt{1-\theta}}{\lambda}\right] \omega^2 \quad (6)$$

where Z_0 is the characteristic impedance of the lossless line. Fig.1 shows the agreement between the values of α obtained using this approximation and the ones from the original PEM expression, in the frequency range from 1–60 GHz, for superconducting microstrip line. The latter has been verified by the comparison with other method [1].

The effect of superconducting attenuation can be modeled by a quadratic filter with the transfer function [6]:

$$H_\alpha(\omega) = e^{-D_\alpha (\omega \zeta)^2}. \quad (7)$$

Where D_α is a dimensionless parameter, defined by analogy with the quadratic dispersion situation:

$$D_\alpha = \left(d^2\alpha / d\omega^2\right)\left(L/2.0\zeta^2\right). \quad (8)$$

In this expression, L is the length of the line, and ζ is the halfwidth of the pulse. The use of the linear filter approach and its validation are discussed in [6]. For a Gaussian pulse, the output of this filter is also Gaussian, and it can be obtained analytically. The amplitude of the signal is reduced by a factor of $(1 + 4D_\alpha)^{1/2}$, and the half-width of the signal is increased by the same factor, as shown in Fig. 2.

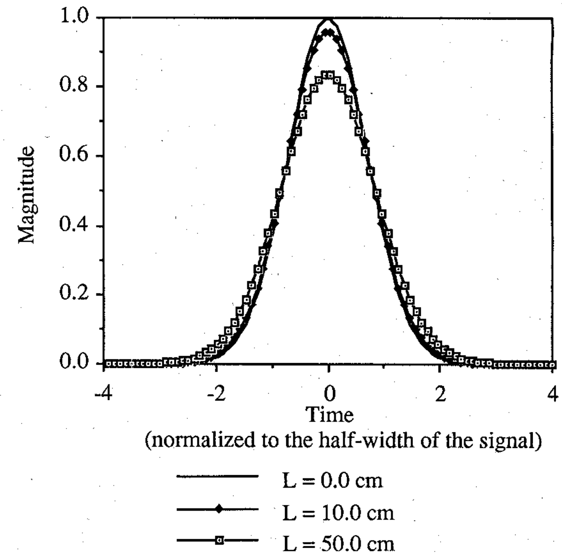

Fig. 2. Attenuation of Gaussian pulse on high T_c superconducting microstrip line.

The simulation in Fig. 2 is for the line described in Fig. 1, with the input Gaussian-shaped pulses of total width of 11 picoseconds and outputs at distances up to 50 cm, values chosen to emphasize the effect of the losses. The attenuation shown in Fig. 2 comes entirely from the conductor loss. The dielectric losses are not included.

As it can be seen from (5a), the predominant term of the internal inductance is linear, so the contribution of the losses to the phase distortion is minimal. The distortion due to the modal dispersion is not considered in this letter; the effects of dispersion will be analyzed in a future work.

It should be emphasized, however, that modal dispersion may play a significant role in the propagation process, which may affect both the magnitude and shape of the signal. The analysis of these effects for conventional lines is found in [7]. Also, for shorter signals, the two-fluid model may not be adequate. Effects of the attenuation and dispersion in low temperature superconducting lines are analyzed in [8].

Conclusion

We have shown that propagation constants for a superconducting microstrip line, obtained by the combination of the two-fluid model and the phenomenological equivalence method, can be approximated by expressions that are consistent with other methods. The attenuation constant is shown to be quadratic on the frequency. The factors involved in these expressions differ from the ones found in the literature in the sense that they include both the geometric factor G and the penetration depth of the superconductor. It was also shown that, within this approximation, the modeling of pulse propagation in such a line is very simple and the attenuation distortion easily estimated. This may provide a new way to characterize superconducting materials from pulse attenuation.

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